

STATUS OF TRI/BIMAXIMAL MIXING

HPS PLB 530 (2002) 167 hep - ph/0202074

(see also HPS PLB 458 (1999) 79 hep - ph/9904297)

Z. Xing PLB B533 (2002) 85 hep - ph/0204049

X. He, A. Zee PLB 560 (2003) 87 hep - ph/0301092

E. Ma hep - ph/0308282

W. SCOTT, RAL, UK

NOVE - II, 3 - 5 Dec 2003

$$U = \begin{matrix} e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

(AT LEAST APPROXIMATELY)

SYMMETRIES OF TBM

$$|U|^2 =$$

$$\begin{array}{c}
 e \\
 \mu \\
 \tau
 \end{array}
 \begin{pmatrix}
 \nu_1 & \nu_2 & \nu_3 \\
 \frac{2}{3} & \frac{1}{3} & 0 \\
 \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
 \frac{1}{6} & \frac{1}{3} & \frac{2}{3} \\
 \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\
 \frac{1}{6} & \frac{1}{3} & \frac{2}{3}
 \end{pmatrix}$$

1) ZERO CP-VIOLATION

(*APPROXIMATE – HOPEFULLY!!*)

HS PLB 535 (2002) 163

hep-ph/0203209

2) $\mu\tau$ 'REFLECTION'

HS PLB 647 (2002) 219

hep-ph/0210197 (*'MAX.'* ~~CP~~)

Grimus/Lavoura hep-ph/0305309

3) ONE TRIMAX. E-VEC.

HS PLB 537 (2003) 76 hep-ph/0302025

SNO Results

Pure D_2O

Q. R. Ahmad et al.

Phys.Rev.Lett. 89 (2002) 011301

nucl - ex / 0204008

$$CC = 1.76 \pm_{0.05}^{0.06} \pm_{0.09}^{0.09}$$

$$NC = 5.09 \pm_{0.43}^{0.44} \pm_{0.43}^{0.46}$$

$D_2O + \text{Salt}$

S. N. Ahmed et al.

Phys.Rev.Lett. ?? (2003) ????

nucl - ex / 0309004

$$CC = 1.70 \pm_{0.07}^{0.07} \pm_{0.10}^{0.09}$$

$$NC = 4.90 \pm_{0.24}^{0.24} \pm_{0.27}^{0.29}$$

(Results quoted here are those given assuming undistorted ^8B - spectrum)

CC/NC

$$= \underline{0.346 \pm_{0.031}^{0.032} \pm_{0.034}^{0.036}}$$

CC/NC

$$= \underline{0.347 \pm_{0.022}^{0.022} \pm_{0.028}^{0.028}}$$

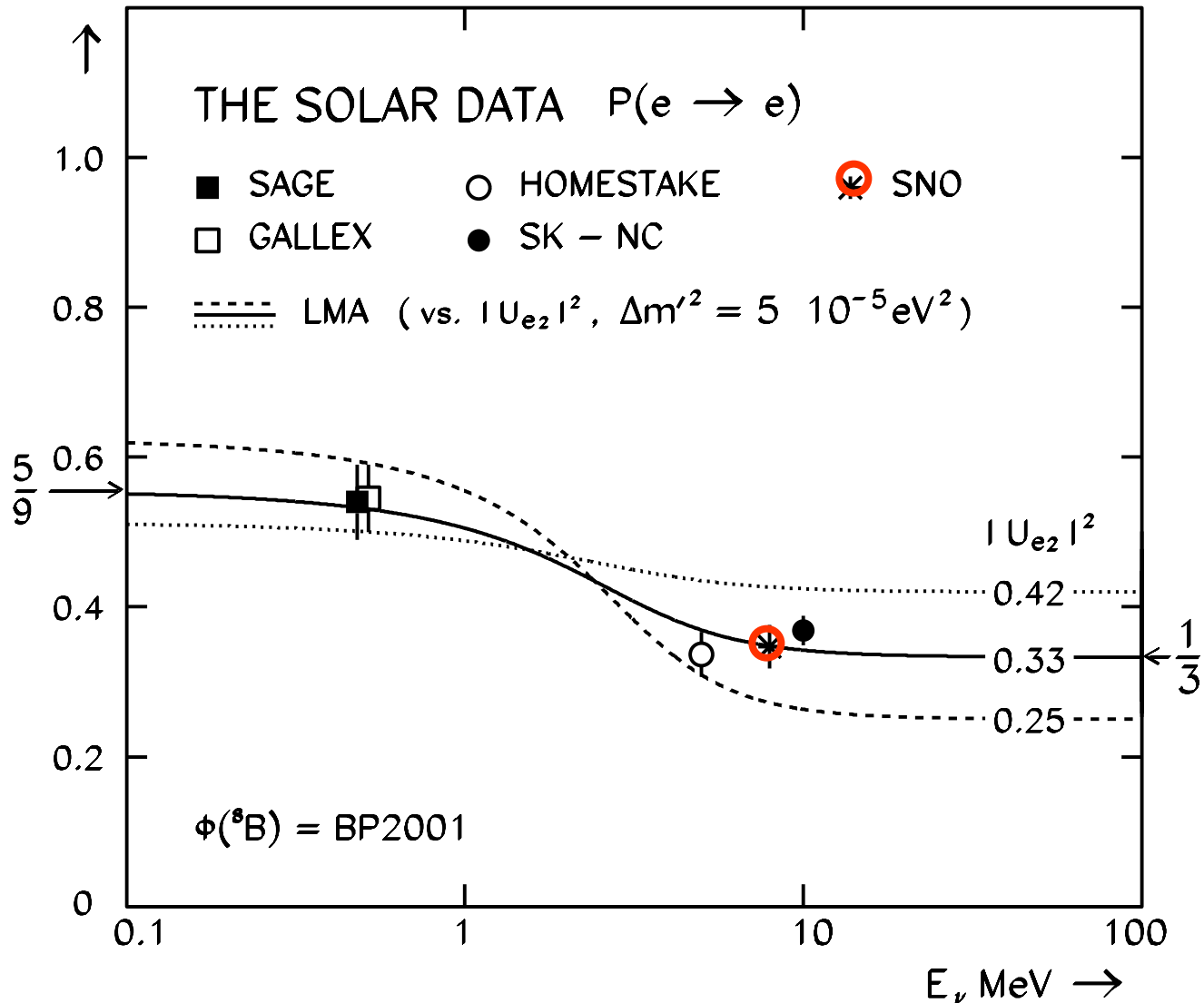
$$\langle CC/NC \rangle \approx \underline{\underline{0.35 \pm 0.03}}$$

(my naive average - ignores correlation in systematic errors)

Solar Data

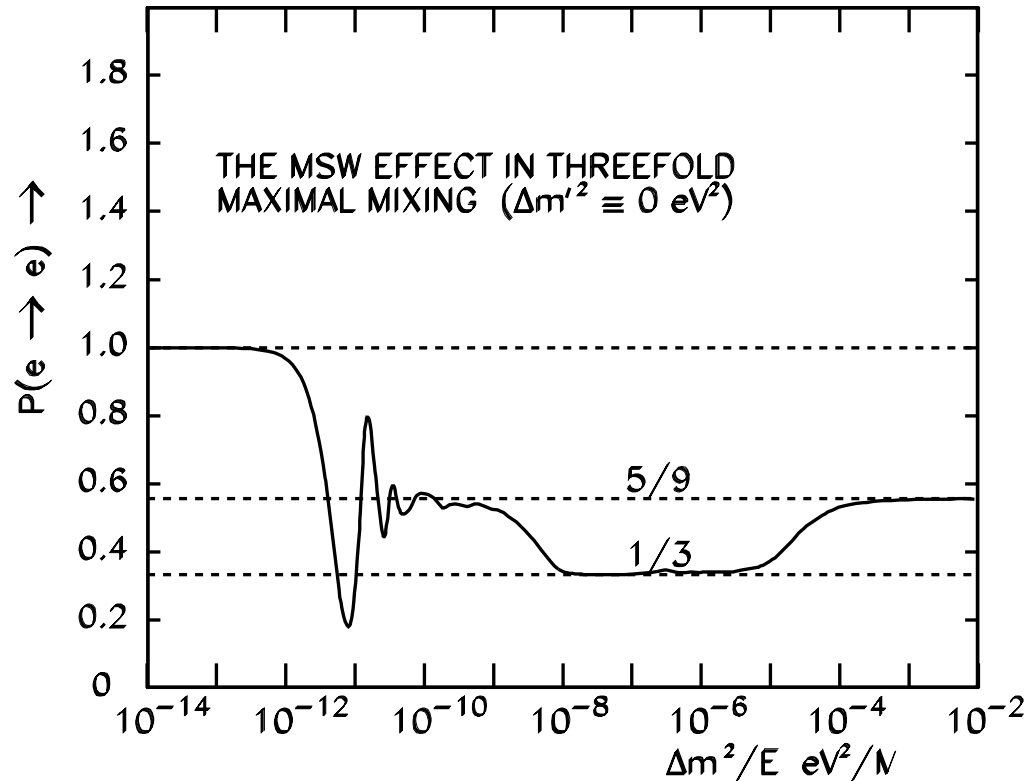
SNO point $CC/NC = 0.35 \pm 0.03$ is my naive average of Salt + No Salt - ignoring corr. syst. and ^8B spect. dist.

HPS PLB 530 (2002) 167 hep-ph/0202074; see also PLB 374 (1996) 111 hep-ph/9601346



THE '5/9-1/3-5/9 BATHTUB'

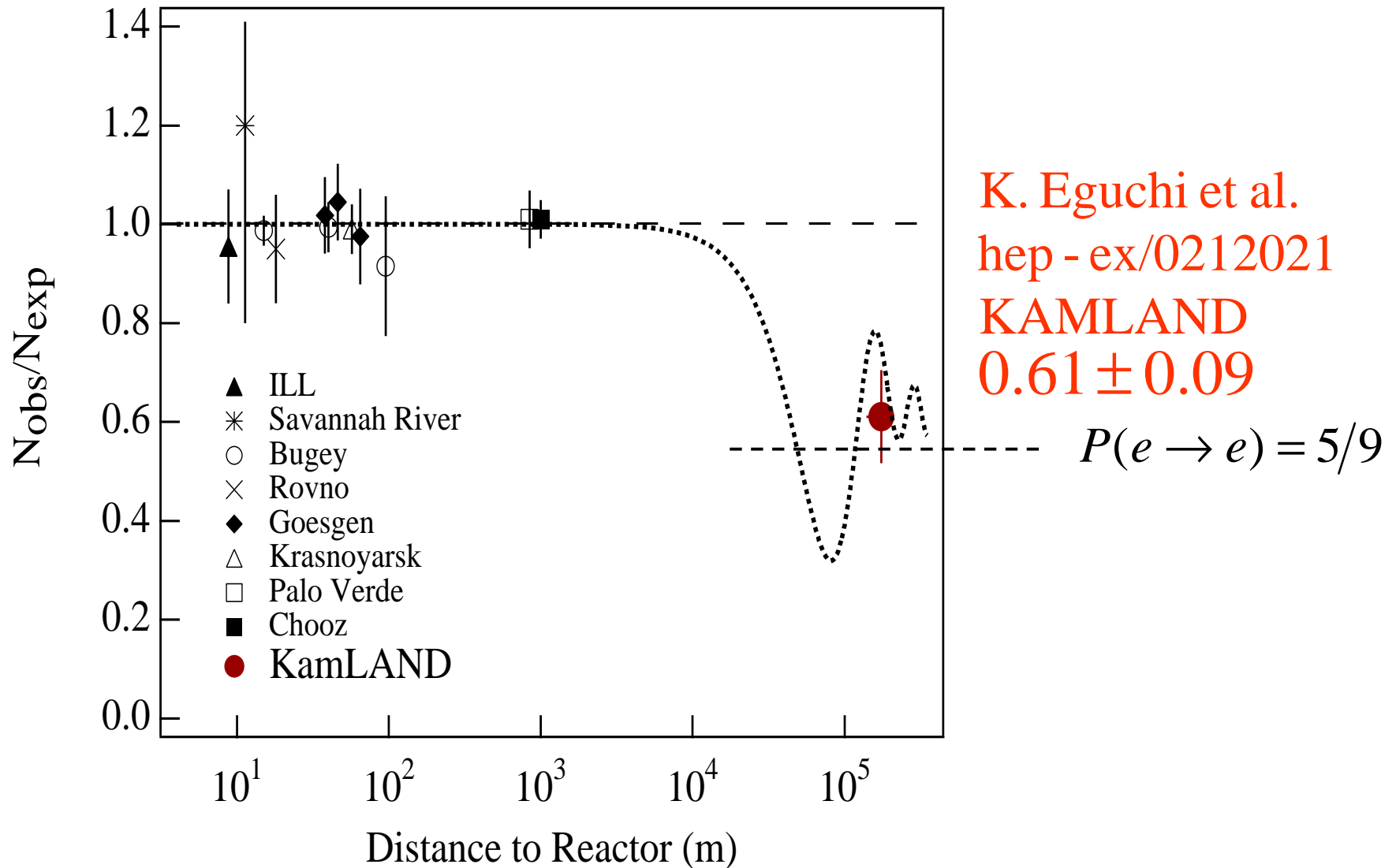
Fig.3 HPS PLB 374 (1996) 111. hep-ph/9601346



PREDICTED IN TRIMAX. MIX.



REACTORS ESP: CHOOZ / PALO - VERDE / KAMLAND



S3 GROUP MIXING

(TRI- $\chi\phi$ MAX. MIXING)

GENERALISES TBM:

$$|U|^2 = \begin{matrix} e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ * & 1/3 & * \\ * & 1/3 & * \\ * & 1/3 & * \end{pmatrix}$$

SYMMETRIC S3 GROUP:

NAT. REP.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P(123) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad P(321) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

$$P(12) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad P(23) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad P(31) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

S3 GROUP MATRIX:

(FLAVOUR BASIS)

NAT. REP.

$$M_{\nu}^2 = aI + bP(123) + b^*P(321) \\ + xP(23) + yP(31) + zP(12)$$

CIRC.

RETRO-CIRC.

$$= \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix} + \begin{pmatrix} x & z & y \\ z & y & x \\ y & x & z \end{pmatrix}$$

3×3
ORTHOGONAL
'MAGIC - SQUARES'

→ TRI-~~χ~~ϕ MAX. MIXING

= S3 GROUP MIXING

S3 GROUP MIXING

(TRI- $\chi\phi$ MAX. MIXING)

GENERALISES TBM:

$$|U|^2 = \begin{matrix} e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ * & 1/3 & * \\ * & 1/3 & * \\ * & 1/3 & * \end{pmatrix}$$

CYCLIC C3 GROUP:

NAT. REP.

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P(123) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad P(321) = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

C3 GROUP MATRIX:

NAT. REP.

$$M^2 = aI + bP(123) + b^*P(321)$$

CIRCULANT

$$= \begin{pmatrix} a & b & b^* \\ b^* & a & b \\ b & b^* & a \end{pmatrix}$$

→ TRIMAX. MIXING

TRI-MAXIMAL MIXING:

HPS PLB 349 (1995) 357

N. Cabibbo PL 72B (1978) 222.

$$U = \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \mu & \frac{\bar{\omega}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\omega}{\sqrt{3}} \\ \tau & \frac{\omega}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{\bar{\omega}}{\sqrt{3}} \end{matrix}$$

(cf. C3 CHARACTER TABLE)

N. Cabibbo:

“ We are probably far from this.... . but not very far...”

Lepton-Photon 2001

NORMALISED

S3 CLASS OPERATORS:

$$c_1 := I \quad c_2 := \frac{(123) + (321)}{\sqrt{2}} \quad c_3 = \frac{(12) + (23) + (31)}{\sqrt{3}}$$

S3 CLASS MULTIPL. TABLE:

	c_1	c_2	c_3
c_1	c_1	c_2	c_3
c_2	c_2	$c_1 + c_2 / \sqrt{2}$	$\sqrt{2}c_3$
c_3	c_3	$\sqrt{2}c_3$	$c_1 + \sqrt{2}c_2$

S3 CLASS ALGEBRA:

NAT. REP.

$$c_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 1/\sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix}$$

$$c_3 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & \sqrt{2} \\ 1 & \sqrt{2} & 0 \end{pmatrix}$$

S3 CLASS MATRIX:

(ν MASS BASIS)

NAT. REP.

$$M_l^2 = pc_1 + qc_2 + rc_3$$

$$= \begin{pmatrix} p & q & r \\ q & p + q/\sqrt{2} & \sqrt{2}r \\ r & \sqrt{2}r & p + \sqrt{2}q \end{pmatrix}$$

→ TRI-BI-MAX. MIXING

S3 CLASS MIXING

(= TRI-BI-MAX. MIXING)

IDENTIFY :

$$q = \sqrt{2} \left(\frac{m_\tau^2 + m_\mu^2}{6} - \frac{m_e^2}{3} \right) \quad r = \frac{m_\tau^2 - m_\mu^2}{2\sqrt{3}}$$

$$p = \frac{m_\tau^2 + m_\mu^2}{6} + \frac{2}{3} m_e^2$$



$$U = \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e & \left(\begin{array}{ccc} \frac{\sqrt{2}}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & 0 \\ 1 & 1 & -1 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{array} \right) \\ \mu & & & \\ \tau & & & \end{matrix}$$

(cf. THE S3 GROUP-CHARACTER TABLE).

$S1 \subset S2 \subset S3$

NAT. REP. $S3$

CLASS OPERATORS:

$$C(1) = I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad C(2) = P(\mu\tau) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

IDENTITY OPERATOR
(`IDENTITY', $I = +1$)

$\mu\tau$ EXCHANGE OPERATOR
(`MUTATIVITY', $M = \pm 1$)

$$P(e\mu) + P(\mu\tau) + P(\tau e) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

DEMOCRATIC MASS – MATRIX
(`DEMOCRACY', $D = 0, 3$)

$S1 \subset S2 \subset S3$

CLAS-MATRIX:

(FLAVOUR BASIS)

NAT. REP. S3

$$M_\nu^2 = s C(1) + t C(2) + u C(3)$$

$$= \begin{pmatrix} s+t+u & u & u \\ u & s+u & t+u \\ u & t+u & s+u \end{pmatrix}$$

→ TRI-BI-MAX. MIXING

(MASS FORMULA ': $m_i^2 = sI_i + tM_i + uD_i$)

S2 \subset S3 MIXING

IDENTIFY : $s = \frac{m_1^2 + m_3^2}{2}$ $t = \frac{m_1^2 - m_3^2}{2}$ $u = \frac{m_2^2 - m_1^2}{3}$



$$U = \begin{matrix} e \\ \mu \\ \tau \end{matrix} \begin{pmatrix} \nu_1 & \nu_2 & \nu_3 \\ \frac{\sqrt{2}}{\sqrt{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} \end{pmatrix}$$

\equiv *TRI - BIMAX.* - *AGAIN !!!*

SUMMARY

1) TBM CONSISTENT WITH THE DATA (ESP. SNO)

2) TBM HAS THREE SYMMETRIES

(ZERO CP, $\mu\tau$ REFLECTION, $\nu_2 = \text{TRIMAX.}$)

3) 'S3 GROUP MIXING' \supset TBM (FLAVOUR BASIS)

$$|U|^2 = \begin{matrix} & \nu_1 & \nu_2 & \nu_3 \\ e & \left(\begin{array}{ccc} * & 1/3 & * \\ * & 1/3 & * \\ * & 1/3 & * \end{array} \right) \\ \mu & & & \\ \tau & & & \end{matrix}$$

4) 'S3 CLASS MIXING' \equiv TBM (ν -MASS BASIS)

5) 'S2 \subset S3 MIXING' \equiv TBM (FLAVOUR BASIS)