Maybe the fitting could help us to find the material properties from the measurements on the wire

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The fitting of the radial stress at the wire centreline

Assuming that stress waves were created in a long cylinder of radius $a$, which was heated uniformly (and instantaneously) to a temperature $\Delta T$ within a central area of radius $r_{in}$, the radial component of dynamic stress can be calculated analytically (see P. Sievers work on this subject). If we simplified our wire-test case (considering 2-D case and forgetting about exponential rise of the current) in order to express the radial stress analytically, we can use the following formula as the most appropriate:

$$\sigma(r, t) = \frac{2E\alpha\Delta T}{1-\nu} \sum_{n=0}^{\infty} \xi_0 \frac{J_1(\epsilon_{0,n}\xi_0)}{J_0^2(\epsilon_{0,n})\epsilon_{0,n}} \left[ J_0(\epsilon_{0,n}\xi) - (1-\nu)\frac{J_1(\epsilon_{0,n}\xi)}{\epsilon_{0,n}} \right] \cos(\epsilon_{0,n}\Theta),$$

(1)

where $E$ is the elastic modulus of the material, $\alpha$ is the coefficient of thermal expansion and $\nu$ is the Poisson’s ratio. $J_1(*)$ and $J_0(*)$ are the corresponding Bessel functions with $\xi_0 = r_{in}/a$ (= 1 in our case), $\xi = r/a$ and $\Theta = ct/a$. $\epsilon_{0,n}$ are the roots of the first-order Bessel functions. The velocity of sound ($c$) is given by:

$$c = \sqrt{\frac{E}{\rho(1-\nu^2)}},$$

(2)

where $\rho$ is a density of the material. In the case of finite rise-time ($t_0$) of the temperature increase, Eq.(1) must be scaled by the factor $1/\Theta_0$, where $\Theta_0 = ct_0/a$.

Figure 1a (black dashed line) shows the LS-Dyna result on stress at the centreline of the 0.6mm diameter wire ($a = 0.3$mm). The maximal current is 8 kA; pulse length ($t_0$) is 600ms. In order to fit this result the Eq.(1) should be used but it is very complicated. On the other hand, it is clear that, after the heat input stops, the stress changes with time in the cos-like (or sin-like) way. So, as an approximation I have used the following parameterization:

$$\sigma = \frac{2 \cdot P_1 \cdot P_2 \cdot P_3}{\Theta_0 \cdot (1 - P_4)} \cdot \cos[P_6 \cdot \pi \cdot \Theta + P_7 \cdot \pi],$$

(3)
with:
\[ c = \sqrt{\frac{P_1}{P_5(1 - P_4^2)}}. \]  
(4)

The result of the fit (the MINUIT program package was used) is shown in Figure 1a (red line). The meaning of the parameters and corresponding values for tantalum are given in Table 1. Please note that numbers in column 'LS-Dyna input' represent 'starting' values of material properties (at 2000K).

Also, first number for temperature rise in that column represents the rise at the centreline of the wire while the second corresponds to the temperature rise at the outer surface.

**Table 1. Tantalum property values used as an input for LS-Dyna and obtained as a result of the fitting procedure. Numbers in parantheses are estimated errors of the parameters (in the last digits).**

<table>
<thead>
<tr>
<th>Par.</th>
<th>Property</th>
<th>LS-Dyna input</th>
<th>fit (stress)</th>
<th>fit (velocity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>El. modulus, ( E ) [GPa]</td>
<td>144</td>
<td>157 (2)</td>
<td>149 (1)</td>
</tr>
<tr>
<td>2</td>
<td>CTE, ( \alpha ) [10^-6 m/m/K]</td>
<td>7.4</td>
<td>6.8 (6)</td>
<td>6.2 (5)</td>
</tr>
<tr>
<td>3</td>
<td>Temp. rise, ( \Delta T )</td>
<td>128-137</td>
<td>123 (10)</td>
<td>119 (6)</td>
</tr>
<tr>
<td>4</td>
<td>Poisson’s ratio, ( \nu )</td>
<td>0.33</td>
<td>0.26 (1)</td>
<td>0.25 (2)</td>
</tr>
<tr>
<td>5</td>
<td>Density, ( \rho ) [g/cm³]</td>
<td>16.0</td>
<td>16.1 (2)</td>
<td>15.6 (2)</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>0.78 (2)</td>
<td>0.80 (5)</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>'Phase'</td>
<td>0.22 (1)</td>
<td>0.71 (1)</td>
<td></td>
</tr>
</tbody>
</table>

**The fitting of the surface velocity**

Unfortunately, we are not able to measure stress but Eq.(1) can help us to find the fitting formula for surface velocity or displacement (see Fig 1b, black dashed line for result on LS-Dyna simulations). By using Hooke’s law (\( \text{strain}= \sigma/E \)) and with a definition for \( \text{strain}= \Delta r/a \), it is clear that in the case of surface velocity \( (v = dr/dt) \) we can use the similar fitting formula as in the previous case:

\[ v = \frac{P_2 \cdot P_3}{(1 - P_4)} \cdot a \cdot t_0 \cdot \cos[P_6 \cdot \pi \cdot \Theta + P_7 \cdot \pi], \]  
(5)
with:

\[ c = \sqrt{\frac{P_1}{P_2(1 - P_1^2)}} \]  

(6)

The result of the fit is shown in Figure 1b (red line).

Figure 1. Radial stress along centreline (a) and surface velocity (b) of the 0.6mm diameter wire calculated by LS-Dyna (black dashed line) and obtained by fitting procedure (red line).
Comments

(1) Equation (1) is an approximation to the real case. Also, the fitting formula is the simplest possible approximation to the Eq.(1) (the simple cosinus function replaces the whole summing term). The fitting function has only seven parameters and meaningful values were obtained for all the parameters. Even the ’phase’ parameter is good. The difference in phase between stress and velocity is exactly π/2 (as it is)! If the temperature rise is fixed to the real value(s) the agreement for other parameters is better (but, my intention was to vary the temperature too).

(2) The simpler (2-D) case was considered but it seems (from the first 3-D LS-Dyna calculations) that 3-D case should not be a problem with the appropriate choice of additional longitudinal ’contribution’ term in the fitting formula. It should be checked on simulations results!

(3) If not for obtaining of the final results for the material parameters, the fitting procedure looks good enough to be used as starting point for LS-Dyna ’tuning’.