4.3 Beam parameters

Canonical variables are of paramount importance in the dynamics of Hamiltonian (i.e. conservative) systems. In these coordinates the volume occupied by an ensemble of particles is an invariant motion. If inter-plane coupling is absent, the areas (i.e. normalized emittances) occupied in the three sub-spaces are also conserved. Position and momentum, time and energy are examples of canonical coordinates. To be precise, it is customary to make a small distinction between area $A$ and emittance $\varepsilon$. Consider an ellipse with semi-axes $a, b$; the area is $A = \pi \varepsilon = \pi ab$. The areas, in the accelerator, of coordinate space available to the emittance ensembles are known as acceptances. Because of the radial sweep in an FFAG, the meaning of transverse acceptance has to be refined: at any momentum there is a guaranteed minimum normalized acceptance.

The over-arching considerations, in falling priority, for scaling between the muon FFAGs and the electron model are (i) that the ratios of acceptance and emittance be comparable; and (ii) that values be selected to facilitate the beam-measurements research objective of the model.

4.3.1 transverse emittance and acceptance

The transverse normalized emittance $\varepsilon^* \Delta x \times \Delta p_\perp / (m_0 c)$ is related to the geometric emittance $\varepsilon_g$ of position and angular divergence by $\varepsilon^* = \varepsilon_g \beta \gamma$. The beam size, a measurable quantity, is given by $\Delta x = \sqrt{\beta \varepsilon_g}$ and will vary during acceleration even if the optics function $\beta$ is constant.

Because the FFAGs have linear fields, the acceptance is limited by the physical aperture (vacuum pipe and any obstructions) rather than the dynamical aperture. In the FFAGs the aperture is taken large enough to contain the radial sweep and the beam size; these depend on the lattice optics (magnet strengths, beam rigidity, etc) which is already chosen. Hence the electron emittance is chosen to have the beam sweep and fill a similar fraction of the aperture; and thereby experience similar levels of field-error and emittance growth as might occur for muons.

4.3.2 longitudinal emittance and acceptance

Energy and time, $\Delta E \times \Delta T$, and also momentum and length $(\Delta p_\parallel / m_0 c) \times \Delta L$ both provide bases for invariant emittances - in units of eV sec and metre, respectively.

Confusingly, perhaps, the intrinsic acceptance of a storage ring is often quoted as a relative momentum bite ($\delta p/p$) in per cent - making it independent of rf harmonic number. However, when acceleration is involved, such sloppiness cannot be afforded; the area of the phase-space manifold (responsible for transport from low to high energy) must be clearly stated.

The longitudinal dynamics of the nonscaling FFAG is strongly nonlinear. To avoid excessive emittance growth, the bunch should occupy only a small fraction of the rf wavelength. Applied to a cascade of accelerators operated with equal rf, and given the constancy of $\delta E \times \delta T$, this implies the absolute energy deviations $\delta E$ are similar. Now, the parameter $a$ refers to acceptance in dimensionless units $x = \omega \delta T$ and $y = \delta E / \Delta E$. Because the energy
ranges, $\Delta E$, fall in the lower energy rings, the acceptance parameter $a$ must rise to embrace the same absolute energy deviation. Thus the muon FFAGs with $\Delta E = [2.5.5.10]$ GeV have $a = [1/5, 1/5, 1/7]$, respectively. REVERSE this argument -it is back to front!

As a starting point, the longitudinal emittance of the electron model is taken such that the bunch occupies a similar fraction of the rf wavelength (but at 1.3 or 3 GHz), and has a similar relative energy spread $\delta E/\Delta E$ as the 10-20 GeV muon FFAG operated with $a = 1/12$. At larger $a$ values, the bunch acts as an increasingly fine probe of the phase-space structure; and, indeed, this argues for a reduction of the emittance below the scale value.

The r.m.s. and total emittance are 0.01 and 0.05 eV.s, respectively, in the muon FFAG. The rf period is 5 ns. The aspect ratio and orientation angle of a matched bunch depend on longitudinal parameters $a, b$. Representative values ($a = 1/12, b = 1/5$) for the 100% emittance in the 10-20 GeV ring are $\delta x = \pm \pi/10$ radian (or $\delta T = \pm 0.5$ ns) and $\delta y = \pm 1\%$ (or $\delta E = 100$ MeV)). The dimensionless forms permit scaling to any frequency (1.3 or 3 GHz) or energy range (10-20 MeV). Scaled to the electron model, the total energy deviation is $\delta E = \pm 0.1$ MeV and the bunch time spread is $\delta T = \pm 77,33$ ps for a 100% emittance of $7.7, 3.3 \mu eV.s$ at 1.3,3 GHZ.

MAKE A GENERAL statement about the longitudinal emittance aspect ratio of the FFAG being very different from that at exit from cooling channel. Mention how aspect ratio increases as $b$ increases. Similar rate of energy gain and absolute energy acceptance implies $\delta p$ increment is similar, and so $a$ diminishes up the cascade.

Make blanket statement that meanings of $\delta E, \delta T$ have a different meaning here than elsewhere in the text.

(at 1.3,3 GHZ)

Two reasons contribute to this: ; and (ii) it is similar between machines of like type but different energy range. That the relative acceptance is similar implies the absolute acceptance ($\delta E$ or $\delta p$) increases along a